UDC 533. 6.011

ON CERTAIN GAS FLOWS IN A GRAVITATIONAL FIELD

PMM Vol. 42, № 1, 1978, pp. 96-104

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(Received May 31, 1977)

New exact solutions are derived for the equations of plane-parallel isentropic flow of gas with polytropic equation of state in a gravitational field. It is shown that when the adiabatic exponent exceeds two these solutions define a mixed flow of supersonic and subsonic streams in infinite channels of special form. An exact solution of the unstable problem of dispersion of gas into vacuum at infinitely increasing speed is obtained.

1. Stable isentropic plane-parallel flows of inviscid gas with polytropic equation of state in a gravitational field are defined by a system of equations of the form [1]

$$u_1 \frac{\partial u_i}{\partial x_1} + u_2 \frac{\partial u_i}{\partial x_2} + \frac{2}{\gamma - 1} c \frac{\partial c}{\partial x_i} + \alpha_i = 0 \quad (i = 1, 2) \tag{1.1}$$
$$u_1 \frac{\partial c}{\partial x_1} + u_2 \frac{\partial c}{\partial x_2} + \frac{\gamma - 1}{2} c \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2}\right) = 0$$

where u_k are components of the velocity vector, c is the speed of sound, and α_k are constants which define action of gravity forces.

Let us consider the potential case in which system (1.1) reduces to a single equation for the velocity potential $\Phi(x_1, x_2)$ and the speed of sound is determined by the Bernoulli integral

$$\Phi_1^2\Phi_{11} + 2\Phi_1\Phi_2\Phi_{12} + \Phi_2^2\Phi_{22} + \alpha_1\Phi_1 + \alpha_2\Phi_2 - (\gamma - 1)\times \tag{1.2}$$
$$[K - \alpha_1 x_1 - \alpha_2 x_2 - {}^1/_2(\Phi_1^2 + \Phi_2^2)](\Phi_{11} + \Phi_{22}) = 0$$
$$(\Phi_i = \partial\Phi / \partial x_i, \ \Phi_{ik} = \partial^2\Phi / \partial x_i \partial x_k, \ K = \text{const})$$

$$c^2 = (\gamma - 1)[K - \alpha_1 x_1 - \alpha_2 x_2 - {}^1/_2(\Phi_1^2 + \Phi_2^2)] \tag{1.3}$$

Applying to (1.2) the Legendre transformation

$$V = \Phi_1 x_1 + \Phi_2 x_2 - \Phi \tag{1.4}$$

we obtain for the new unknown function $V(u_1, u_2)$ $(\Phi_1 = u_1, \Phi_2 = u_2)$ the Monge—Ampere equation

$$u_1^2 V_{22} - 2u_1 u_2 V_{22} + u_2^2 V_{11} + (\alpha_1 u_1 + \alpha_2 u_2)(V_{11}V_{22} - V_{12}^2) - \tag{1.5}$$
$$(\gamma - 1)[K - \alpha_1 V_1 - \alpha_2 V_2 - {}^1/_2(u_1^2 + u_2^2)](V_{11} + V_{22}) = 0$$

The solution is reestablished in terms of coordinates x_1 and x_2 by formulas

$$V_1 = x_1, \ V_2 = x_2 \ (V_k = \partial V / \partial u_k, \ V_{ik} = \partial^2 V / \partial u_i \partial u_k) \tag{1.6}$$

We seek for Eq. (1.5) the class of exact solutions in the form of a third power polynomial in u_1 and u_2

$$V(u_1, u_2) = \frac{Ku_1}{2\alpha_1} + \frac{Ku_2}{2\alpha_2} + \sum_{0 \leqslant i+k \leqslant 3} a_{ik} u_1^i u_2^k, \quad a_{ik} = \text{const} \tag{1.7}$$

The constant a_{00} in (1.7) is unimportant and for the remaining a_{ik} we obtain from (1.5) a nonlinear system of ten equations with nine unknown a_{ik} $(0 < i + k \leqslant 3)$ and parameters α_1, α_2, and γ. Without affecting the generality we set $\alpha_1 = \alpha_2 = \alpha$ (the force of gravity acts along the bisectrix of the first coordinate angle). The system of four equations for the determination of the leading coefficients a_{30}, a_{12}, a_{21}, and a_{03} is independent (it does not contain remaining a_{ik}). By setting

$$6\alpha a_{30} = y_1, \; 6\alpha a_{03} = y_2, \; 2\alpha a_{12} = z_1, \quad 2\alpha a_{21} = z_2 \tag{1.8}$$

that system can be expressed in the form

$$\begin{gathered}\frac{\gamma+1}{2} y_1 y_2 + \gamma y_1 z_1 + (\gamma-1) z_1^2 + \frac{3}{2}(\gamma-1) y_1 z_2 + \frac{\gamma-3}{2} z_2^2 + \\ \frac{\gamma-1}{2} y_2 z_2 + (\gamma-2) z_1 z_2 + \frac{\gamma+1}{2} y_2 + \frac{\gamma-5}{2} z_2 = 0 \quad (1 \leftrightarrow 2) \\ \frac{\gamma-1}{2} y_1^2 + \frac{\gamma+1}{2} y_1 z_1 + \frac{\gamma-1}{2} y_1 z_2 - z_2^2 + \frac{\gamma-1}{2} z_1 z_2 + \\ \frac{\gamma-1}{2} y_1 + \frac{\gamma+1}{2} z_1 = 0 \quad (1 \leftrightarrow 2)\end{gathered} \tag{1.9}$$

where the symbol $1 \leftrightarrow 2$ indicates that the remaining equations are obtained by the cyclic permutation of subscripts 1 and 2.

The problem of determining leading coefficients of representation (1.7) is thus reduced to the problem of determining all points of intersection of for second order surfaces defined by system (1.9) in the four- dimensional space y_1, y_2, z_1, z_2.

We shall present a complete analysis of solutions of system (1.9).

First, we carry out the linear transformation of variables in (1.9) (rotation of coordinates) by introducing new variables ξ_1, ξ_2, η_1, and η_2 by formulas

$$\xi_1 = y_1 - y_2, \; \xi_2 = z_1 - z_2, \; \eta_1 = y_1 + y_2, \; \eta_2 = z_1 + z_2 \tag{1.10}$$

In these coordinates system (1.9) assumes the form

$$\begin{gathered}F_1^{\pm} = \frac{\gamma-1}{2}\xi_1^2 + \xi_1\xi_2 - \frac{\gamma+1}{2}\xi_2^2 \pm (\gamma-1)\xi_1\eta_1 + \frac{\gamma-1}{2}\eta_1^2 \pm \\ \gamma\xi_1\eta_2 + \frac{\gamma-3}{2}\eta_2^2 \pm \xi_2\eta_1 \pm 2\xi_2\eta_2 + \gamma\eta_1\eta_2 \pm (\gamma-1)\xi_1 \pm \\ (\gamma+1)\xi_2 + (\gamma-1)\eta_1 + (\gamma+1)\eta_2 = 0 \\ F_2^{\pm} = -\frac{\gamma+1}{2}\xi_1^2 + \xi_1\xi_2 + \frac{\gamma-1}{2}\xi_2^2 + \frac{\gamma+1}{2}\eta_1^2 \pm (2\gamma-1)\xi_1\eta_2 + \\ \frac{5\gamma-9}{2}\eta_2^2 \pm (2-\gamma)\xi_2\eta_1 \pm (\gamma+1)\xi_2\eta_2 + (3\gamma-2)\eta_1\eta_2 \mp \\ (\gamma+1)\xi_1 \mp (\gamma-5)\xi_2 + (\gamma+1)\eta_1 + (\gamma-5)\eta_2 = 0\end{gathered} \tag{1.11}$$

Composing the linear combinations $4(F_1^+ - F_2^-) = 0$ and $4(F_1^- - F_2^+) = 0$, we obtain

$$\xi_1 [(\gamma - 1)\eta_1 + \gamma\eta_2 + \gamma - 1] + \xi_2 (\eta_1 + 2\eta_2 + \gamma + 1) = 0 \qquad (1.12)$$

$$\xi_1 [(2\gamma - 1)\eta_2 - \gamma - 1] + \xi_2 [(2 - \gamma)\eta_1 + (\gamma + 1)\eta_2 - \gamma + 5] = 0$$

We shall now consider two cases:

1°. $(\xi_1, \xi_2) = (0, 0)$, and 2°. $(\xi_1, \xi_2) \neq (0, 0)$.

In case 1° we obtain for the unknown η_1 and η_2 from the equations $F_1^+ = 0$ and $F_1^- = 0$ the system

$$F_1^{+\prime} = \frac{\gamma-1}{2}\eta_1^2 + \frac{\gamma-3}{2}\eta_2^2 + \gamma\eta_1\eta_2 + (\gamma-1)\eta_1 + (\gamma+1)\eta_2 = 0 \qquad (1.13)$$

$$F_1^{-\prime} = \frac{\gamma+1}{2}\eta_1^2 + \frac{5\gamma-9}{2}\eta_2^2 + (3\gamma-2)\eta_1\eta_2 + (\gamma+1)\eta_1 + (\gamma-5)\eta_2 = 0$$

The linear combination $(\gamma + 1)F_1^{+\prime} - (\gamma - 1)F_1^{-\prime} = 0$ yields the relationship

$$\eta_2 [(\gamma^2 - 3\gamma + 3)\eta_2 + (\gamma^2 - 3\gamma + 1)\eta_1 - 2(2\gamma - 1)] = 0 \qquad (1.14)$$

Equating in (1.14) the second term to zero, from Eqs. (1.13) we obtain for η_2 a quadratic equation. Its solution in case 1° presents the following possibilities:

$$\eta_1 = -2, \quad \eta_2 = 0 \qquad (1.15)$$

$$\eta_1 = \frac{(2\gamma-1)(2-\gamma)}{(\gamma-1)^2}, \quad \eta_2 = \frac{\gamma(2\gamma-1)}{(\gamma-1)^2} \qquad (1.16)$$

$$\eta_1 = -\frac{3+\gamma}{3-\gamma}, \quad \eta_2 = \frac{\gamma-1}{3-\gamma} \quad (\gamma \neq 3) \qquad (1.17)$$

and in case 2° from (1.12) we have

$$\frac{\xi_1}{\xi_2} = -\frac{\eta_1 + 2\eta_2 + \gamma + 1}{(\gamma-1)\eta_1 + \gamma\eta_2 + \gamma - 1} \qquad (1.18)$$

with η_1 and η_2 related by the formula

$$(\gamma - 1)(\gamma - 2)(\eta_2^2 - \eta_1^2) + 2(-\gamma^2 + 5\gamma - 3)\eta_1 + 2(-\gamma^2 + 3\gamma - 1)\eta_2 + 4(2\gamma - 1) = 0 \qquad (1.19)$$

The case in which the numerator and the denominator vanish in (1.18) when

$$\eta_1 = \frac{\gamma^2 - \gamma + 2}{\gamma - 2}, \quad \eta_2 = \frac{\gamma - \gamma^2}{\gamma - 2} \quad (\gamma \neq 2) \qquad (1.20)$$

yields after the analysis of compatibility of Eqs. (1.11) $\gamma = 1$ or $\gamma = 1/2$. Hence this case can be eliminated.

If in (1.19) $\gamma = 2$, then from Eqs. (1.11) for η_1 and η_2 we have only the possibility (1.15) but, then, $\xi_1 = \xi_2 = \xi$, where ξ is an arbitrary number. Because of this we consider below (1.19) for $\gamma \neq 2$. The second equation relating η_1 and η_2 is obtained from (1.11), taking into account that in (1.11) the sum of all terms containing first powers of ξ_1 and ξ_2 are zero owing to (1.12). Then from

Eqs. (1.11) for $F_1^+ = 0$ and $F_1^- = 0$, follow relation ($F_1^{+'}$ and $F_1^{-'}$ which are defined in (1.13)) we have

$$\frac{(\gamma-1)\xi_1^2 + 2\xi_1\xi_2 - (\gamma+1)\xi_2^2}{-(\gamma+1)\xi_1^2 + 2\xi_1\xi_2 + (\gamma-1)\xi_2^2} = \frac{F_1^{+'}}{F_1^{-'}} \tag{1.21}$$

which with the use of (1.18) we reduce to the form

$$\begin{aligned}&\gamma(\gamma-1)^2\eta_1^3 + (\gamma-1)(5\gamma^2-5\gamma-2)\eta_1^2\eta_2 + (\gamma-1)^2 \times \\ &(7\gamma-4)\eta_1\eta_2^2 + (\gamma-1)(3\gamma^2-7\gamma+6)\eta_2^3 + 2\gamma(\gamma-1)\times \\ &(\gamma-2)\eta_1^2 + 4(\gamma^3-4\gamma^2+\gamma+1)\eta_1\eta_2 + 2(\gamma^3-5\gamma^2+ \\ &4\gamma-6)\eta_2^2 - 4\gamma(\gamma-1)\eta_1 - 4\gamma(\gamma+1)\eta_2 = 0\end{aligned} \tag{1.22}$$

Then, setting $\eta_1 + \eta_2 = q$ and using (1.19) for representing η_1 and η_2 in the form

$$\begin{aligned}\eta_{1,2} = \{&(\gamma-1)(\gamma-2)q^2 \pm 2[(\gamma\pm1)(2-\gamma) + 2\gamma - 1\, q \pm \\ &4(2\gamma-1)\}\{2(\gamma-2)[(\gamma-1)q-2]\}^{-1}\end{aligned} \tag{1.23}$$

we obtain from (1.22) a fifth power equation for the determination of q. Owing to its unwieldiness that equation is not presented here. Its roots are of the form

$$q_1 = -2, \quad q_2 = \frac{2}{\gamma-2}, \quad q_3 = q_4 = q_5 = \frac{2}{\gamma-1} \tag{1.24}$$

The case of $q = q_{3,4,5}$ when denominators in (1.12) vanish is of no interest, since it follows directly from (1.19) that $\gamma = 2$. When $q = q_1 = -2$ we have

$$\eta_1 = -2, \ \eta_2 = 0, \ \xi_1 = \xi_2 = \xi \tag{1.25}$$

where ξ' is an arbitrary number.

The case of $q = q_2$ corresponds to the already considered possibility (1.20).

Thus for the leading coefficients (1.8) it is necessary to consider the three possibilities (1.25), (1.16), and (1.17).

We pass to the analysis of the system for coefficients at quadratic terms in (1.7). It follows from (1.6) that it is always possible to obtain $a_{10} = a_{01} = 0$ by a suitable transfer of the coordinate origin. Setting

$$2\alpha a_{20} = \mu, \ 2\alpha a_{02} = \nu, \ \alpha a_{11} = \lambda \tag{1.26}$$

it is possible to have a system of equations which must be satisfied by μ, ν, and λ of the form

$$\begin{aligned}&\mu\nu - \lambda^2 + (\gamma-1)(\mu+\lambda)(\mu+\nu) = 0 \ (\mu\leftrightarrow\nu), \quad y_2\nu + z_1\mu - \\ &2z_2\lambda + 2\nu + (\gamma-1)[{}^1/_2(y_2+z_2+1)(\mu+\nu) + \\ &(y_2+z_2)(\mu+\lambda)] = 0 \ (1\leftrightarrow2, \mu\leftrightarrow\nu), \ y_1\mu + z_2\nu - 2z_2\lambda + y_2\nu - \\ &2z_2\lambda - 2\lambda + (\gamma-1)[(z_1+z_2)(\mu+\nu) + (y_1+z_2) + \\ &(\mu+\lambda) + (y_2+z_1)(\nu+\lambda)] = 0\end{aligned} \tag{1.27}$$

System (1.27) has the solutions $\mu = \nu = \lambda = 0$, and its simple analysis shows that there are no other solutions for all y_k and z_k, determined by (1.10) in terms of ξ_k and η_k, that correspond to (1.25), (1.16), and (1.17) when $\gamma > 1$. Hence only third power terms remain in formulas (1.7) and their coefficients are determined either by (1.25), (1.16), or (1.17).

2. Let us consider the physical meaning of the derived variants of the solution of Eq. (1.5). For each of these solutions to have a real meaning it is necessary to establish:

1) the positiveness of the right-hand side of formula (1.3) which defines the square of the speed of sound;

2) the possibility of inversion of formulas (1.6), i.e. the possibility of passing from the hodograph plane in which the solutions are derived, to the physical plane x_1, x_2;

3) the absence of limiting lines in the flow field.

Let us consider all three solutions from these aspects and establish related flow patterns by determining their streamlines and lines of constant density of gas.

In the case of (1.25)

$$a_{30} = \frac{\xi - 2}{12\alpha}, \quad a_{03} = -\frac{\xi + 2}{12\alpha}, \quad a_{12} = a_{21} = \frac{\xi}{4\alpha} \tag{2.1}$$

Using (1.3) and (1.6) we find that the speed of sound is identically zero, hence this possibility must be rejected.

In the case of (1.16)

$$a_{30} = a_{03} = \frac{(2\gamma - 1)(2 - \gamma)}{12\alpha(\gamma - 1)^2}, \quad a_{12} = a_{21} = \frac{\gamma(2\gamma - 1)}{4\alpha(\gamma - 1)^2} \tag{2.2}$$

and from (1.3) we obtain

$$c^2 = -\frac{\gamma}{2(\gamma - 1)}[\gamma(u_1^2 + u_2^2) + 2(2\gamma - 1)u_1u_2] \tag{2.3}$$

which shows that in the hodograph plane the lines of constant speed of sound are hyperbolas with asymptotes

$$u_2 = \eta_\pm u_1, \quad \eta_\pm = \gamma^{-1}(1 - 2\gamma \pm \sqrt{(\gamma - 1)(3\gamma - 1)}) \tag{2.4}$$

Since the discriminant of the trinomial in (2.3) is positive for all γ, the right-hand side of (2.3) is nonnegative in the sectors contained in the second and fourth quadrants formed by the asypmtotes (2.4) in the plane u_1, u_2.

Calculating the Jacobian J by formula (1.3) we obtain

$$J = \frac{D(x_1, x_2)}{D(u_1, u_2)} = -\frac{(2\gamma - 1)^2}{2(\gamma - 1)^3\alpha^2}(\gamma u_1^2 + 2u_1u_2 + \gamma u_2^2) \leqslant 0$$

Consequently, the transition to the x_1, x_2 -plane is always possible and there are no limiting lines in the flow ($J = 0$ only at point $(u_1, u_2) = (0, 0)$).

The transition to the physical plane x_1, x_2 is effected by formulas

$$x_i = \frac{K}{2\alpha} + \frac{2\gamma - 1)}{2\alpha(\gamma - 1)^2} f_i(u_1, u_2) \quad (i = 1, 2) \tag{2.5}$$

$$f_1(u_1, u_2) = f_2(u_2, u_1) = \frac{2 - \gamma}{2}u_1^2 + \gamma u_1u_2 + \frac{\gamma}{2}u_2^2$$

Since the constant K in (2.5) is immaterial, we set $K = 0$.

Integration of the equation of streamlines

$$dx_1/u_1 = dx_2/u_2$$

yields for these the following parametric formulas:

$$x_i = C_\lambda \left|1-u\right|^\alpha \left|u^2+\frac{2(2\gamma-1)}{\gamma}u+1\right|^\beta f_i(1,u) \quad (i=1,2) \tag{2.6}$$
$$\alpha = -\frac{2(\gamma-1)}{3\gamma-1}, \quad \beta = -\frac{2\gamma}{3\gamma-1}$$

where the parameter $u \in (\eta_-, \eta_+)$, and for a particular streamline $C_\lambda = \text{const}$. Along the straight lines

$$x_2 = \eta_\pm x_1 \tag{2.7}$$

the speed of sound is zero and the streamlines asymptotically approach these with $u \to \eta_\pm$.

Streamlines l_1 and l_2, and asypmtotes $S_\pm$ are shown in Fig. 1 in the hodograph and the x_1, x_2 planes for $\gamma = 1.4$ and $\gamma = 3$ by solid and dash lines, respectively.

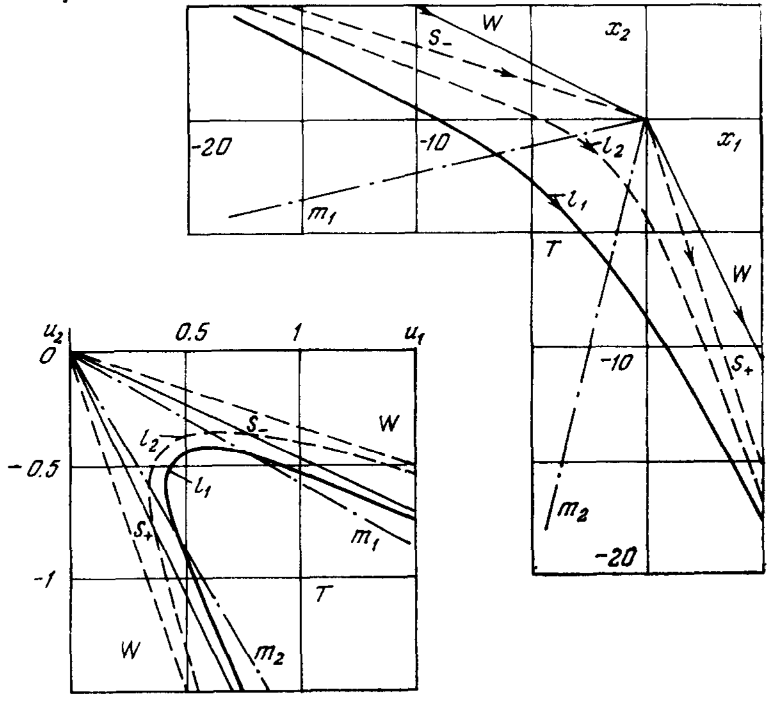

Fig. 1

The flow region T comprises the whole of the third quadrant (density indefinitely increases along the quadrant bisectrix with increasing distance from the coordinate origin), and is bounded by segments of asymptotes $S_\pm$ that separate the flow field from the vacuum zone W. When $\gamma < 2$ the flow is supersonic throughout T, while for $\gamma \geqslant 2$ two sonic lines m_1 and m_2 appear in it (merging for $\gamma = 2$ at the bisectrix $x_1 = x_2$) so that the supersonic stream rising from infinity against the force of gravity is decelerated, becomes subsonic after crossing line m_1 and, then, after crossing line m_2 is accelerated and becomes again supersonic.

In the case of (1.17) we have

$$a_{30} = a_{03} = -\frac{3+\gamma}{12\alpha(3-\gamma)}, \quad a_{12} = a_{21} = \frac{\gamma-1}{4\alpha(3-\gamma)}$$
$$c^2 = \frac{(\gamma-1)^2}{2(3-\gamma)}(u_1-u_2)^2 \qquad (2.8)$$
$$J = -\frac{\gamma^2-1}{2(3-\gamma)\alpha^2}\left(u_1^2 - \frac{4}{\gamma-1}u_1u_2 + u_2^2\right)$$

Hence γ needs to be considered only in the range $1 < \gamma < 3$. Formulas for passing to the x_1, x_2 -plane are of the form

$$x_i = \frac{g_i(u_1, u_2)}{2\alpha(3-\gamma)} \quad (i=1,2) \qquad (2.9)$$
$$g_1(u_1, u_2) = g_2(u_2, u_1) = -\frac{3+\gamma}{2}u_1^2 + (\gamma-1)u_1u_2 + \frac{\gamma-1}{2}u_2^2$$

The lines of constant speed of sound $c = \delta$ are straight lines in the hodograph plane and parabolas

$$x_1 + x_2 = -\frac{1}{\delta^2}\frac{(3-\gamma)^3}{(\gamma+1)^2}(x_1-x_2)^2 - \frac{3-\gamma}{\gamma+1}\delta^2$$

in the x_1, x_2 -plane.

The second of Eqs. (2.9) implies that along the straight lines

$$u_2 = p_\pm u_1, \quad p_\pm = (\gamma-1)^{-1}\left(2 \pm \sqrt{(3-\gamma)(\gamma+1)}\right) > 0 \qquad (2.10)$$

the Jacobian J vanishes, and the equations of streamlines are of the form

$$x_i = D_\lambda |1-u|^{\omega_+}\left|u^2 + \frac{4\gamma}{\gamma-1}u + 1\right|^{\omega_-} g_i(1, u) \quad (i=1,2) \qquad (2.11)$$
$$\omega_\pm = -\frac{\gamma\pm1}{3\gamma-1}, \quad u \in (u_-, u_+), \; D_\lambda = \text{const}$$
$$u_\pm = (\gamma-1)^{-1}\left(-2\gamma \pm \sqrt{(3\gamma-1)(\gamma+1)}\right) < 0$$

There are also two straight streamlines

$$x_2 = u_\pm x_1 \qquad (2.12)$$

to which all streamlines asymptotically approach when $u \to u_\pm$. The speed of sound, contrary to the case of (1.16), indefinitely increases with increasing distance from the coordinate origin.

To determine the flow as a whole without limit lines it is necessary to consider as the region of flow in the hodograph plane, the sectors in the second or fourth quadrants bounded by the straight lines

$$u_2 = u_\pm u_1 \qquad (2.13)$$

By virtue of (2.10) the Jacobian J does not vanish within these sectors and the passing to the x_1, x_2 -plane using (2.9) is unambiguous. In the x_1, x_2 -plane the straight line $x_1 = x_2$ corresponds to the bisectrix of sectors $(u_1 + u_2 = 0)$. The speed of sound indefinitely increases with the increasing distance along that line from the coordinate origin when $x_1 = x_2 < 0$.

The asymptotes (2.12) and (2.13) and two streamlines l_1 and l_2 which form a channel in which flows a supersonic stream of gas are plotted in Fig. 2 in the hodograph and the x_1, x_2 -planes for $\gamma = 1.4$. As in the case of (1.16), for $2 \leqslant \gamma < 3$ a subsonic zone bounded by two sonic lines (that for $\gamma = 2$ merge with the line $x_1 = x_2$) and containing the bisectrix $x_1 = x_2$ appears in the flow.

Note. If the sectors bounded by the straight lines (2.3) in the hodograph planes which encompass the first or third quadrants are used for defining the flow region in the hodograph plane, it is not possible to construct a flow which, as a whole, is free of limit lines.

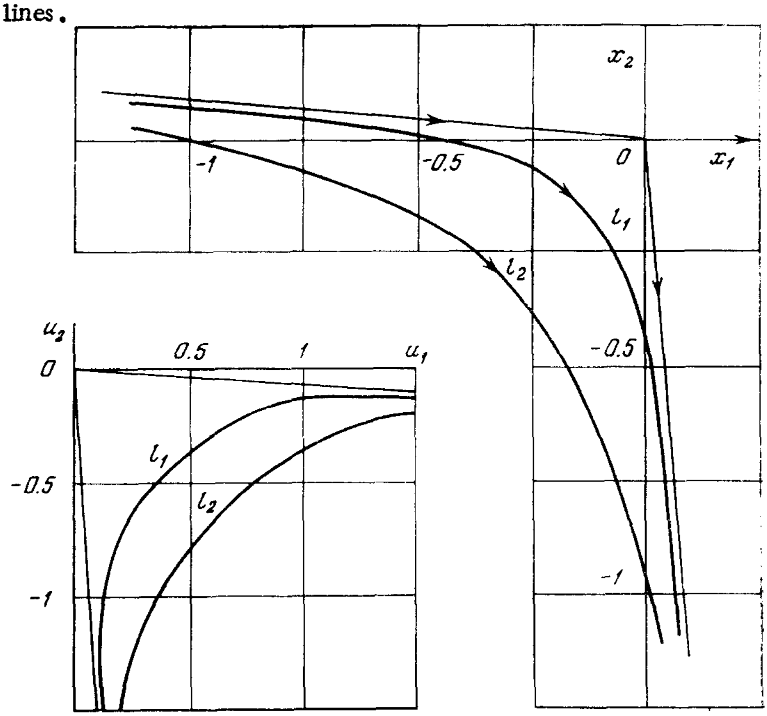

Fig. 2

3. The flow of gas defined by formulas (2.3) and (2.5), whose particular cases are shown in Fig. 1 are determinate in the whole plane $|x_k| < \infty$, which contains the regions of vacuum W and of flow T. We shall derive the solution of the following problem. Let the steady flow (2.3), (2.5) in a gravity field define the input data of the Cauchy problem in the plane $|x_k| < \infty$ for the unsteady solutions of gasdynamics with independent variables x_1, x_2, and t in the absence of mass forces. Solution of that Cauchy problem corresponds to that of the problem of unsteady scatter of gas from region T into vacuum with the gravity field instantaneously cancelled at the instant of time $t = 0$

We set
$$\xi_k = x_k - \frac{\alpha}{2} t^2, \quad \mathbf{V}(x_1 x_2, t) = \mathbf{u}(\xi_1, \xi_2) + \boldsymbol{\alpha} t \tag{3.1}$$

where $\boldsymbol{\alpha} = (\alpha, \alpha)$, $\mathbf{u} = (u_1, u_2)$ and $\mathbf{u}(\xi_1, \xi_2)$ is calculated by formulas (2.5) in which ξ_k is substituted for x_k. We determine the speed of sound by the formula

$$c^2(\xi_1, \xi\ ) = (\gamma - 1)[K - \alpha\xi_1 - \alpha\xi_2 - {}^1/_2(u_1^2(\xi_1, \xi_2) + u_2^2(\xi_1, \xi_2))] \tag{3.2}$$

Then formulas (3.1) and (3.2) provide the solution of the formulated Cauchy problem that is definite for all $t \in (0, \infty)$ and $|x_k| < \infty$. The unsteady equations of gasdynamics are satisfied in the absence of mass forces because the introduced system of coordinates ξ_1, ξ_2 moves with constant acceleration along the bisectrix of the first angle of coordinates. Since for $t = 0$ $\xi_k = x_k$, $\mathbf{V} = \mathbf{u}(x_1, x_2)$ and $c(\xi_1, \xi_2) = c(x_1, x_2)$, the stated initial data of the Cauchy problem are satisfied. The obtained solution corresponds to a rarefaction at all t. The discharge front into vacuum from region T is formed by two planes

$$x_2 = \eta_{\pm} x_1 + \frac{a}{2\gamma}\sqrt{3\gamma-1}\left(\sqrt{3\gamma-1} \mp \sqrt{\gamma-1}\right) t^2$$

that intersect at $t \geqslant 0$ on the bisectrix of the first quadrant. The rate of discharge into vacuum indefinitely increases with the increase of t.

The author thanks O. B. Khairullina for assistance in checking calculations and carrying out computations.

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